HOT WATER FLOODING OF OIL-BEARING BEDS UNDER CONDITIONS OF MUTUAL SOLUBILITY OF WATER AND CRUDE

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In large-scale approximation this article investigates the unidimensional process of displacement of oil by hot water at high pressures and temperatures, when water and oil are partly or fully mutually soluble.

Displacement of oil by hot water, where the thermal effect is utilized for attaining partial or full mutual solubility of water and oil, is a new trend in the development of thermal methods of increasing the oil yield of beds [1, 2]. The possibility of effecting such a regime of displacement has now been experimentally proved [2]. The main complications of its being put into effect are due to the fact that mutual solubility of water and oil occurs at high pressures and temperatures. For instance, for degassed oil of the Dolinsk deposit with a viscosity of 45 mPa·sec full mutual solubility is attained at a pressure of 21.6 MPa and a temperature of 338°C [1]. This also determines the object of the effect (the bed must lie at a depth of at least 1800 m) and the requirements the equipment has to fulfill. Moreover, the mutual solubility of water and oil drops drastically when the temperature drops below the critical temperature T_C corresponding to full mutual solubility, and therefore another complicating circumstance is the heat exchange between the bed and the surrounding rocks.

Below, a hydrodynamic analysis of the mechanism of displacement of oil by hot water with partial or full mutual solubility of water and oil is presented [3]. The process is examined in large-scale approximation [4]: capillary and nonequilibrium effects and heat conduction along the bed are not taken into account so that the heat spreads only by convection, and the temperature of the liquids and of the rock coincide at each point [5-7]. The bed is assumed to be thin, the heat flux through its roof and floor is proportional to the difference between the temperatures of the bed T and of the surrounding rocks T_0 = const, in addition, thermal expansion of the liquids and the effect of a change of pressure on the phase equilibrium of water and oil are neglected. The solution is plotted by the method of characteristics [8, 9]. It was established that there exist two regimes of displacement. One of them, the pseudopiston regime, is effected at higher temperature T° of the hot water pumped into the bed; it is characterized by the fact that there is no zone of mixing of water and oil in the structure of the displacement front. The second regime is associated with lower temperature T°, and the advance of the thermal front is accompanied by the mixing of water and oil. Heat exchange between the bed and the surrounding rocks leads to the transition from the first regime of displacement to the second regime, and the temperature of transition T°_* is lower than T_C. This effect is analogous to the known effect in the theory of displacement of oil by solvents [10]. However, in our case T°_* is not only determined by the conditions of phase equilibrium of water and oil but also by the dependence of the hydrodynamic characteristics of the flow on the temperature.

Figure 1a shows a typical phase diagram of the system water—oil at constant pressure, the variables being full volume water content σ—temperature T. The binodal curve $T = T^*(\sigma)$ separates the regions of single-phase Δ' and two-phase Δ'' states of the system. The intersections of the family of horizontal nodal curves with the binodal curve determine the volume concentrations of water in the aqueous phase w(T) and in the oil phase ψ(T) at the given temperature; C is the critical point at which $w(T_C) = \psi(T_C)$ and $dT^*/d\sigma = 0$. Along each nodal curve, e.g., $T = T_2^\circ$ and $\psi(T_2^\circ)\leqslant\sigma\leqslant w(T_2^\circ)$, the compositions of the aqueous and oil phases are invariable, and their relative volumes (s and 1 — s) are inversely proportional to the lengths of the line segments, e.g., D_1D_W and D_1D_ψ, into which the mapping point $D_1 = (\sigma, T_2^\circ)$ divides the nodal curve $D_\psi D_W$.

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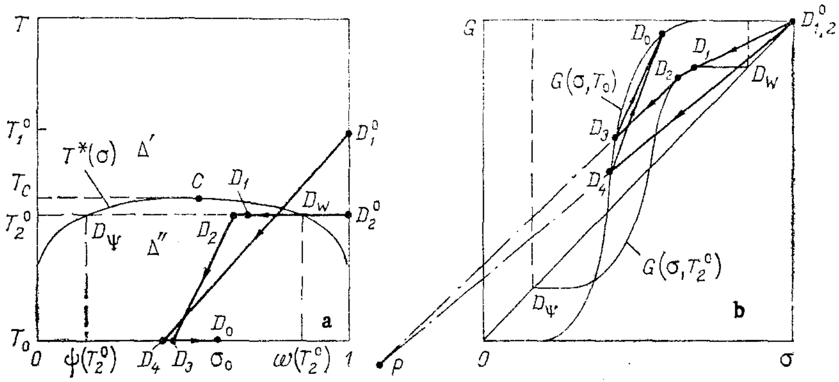

Fig. 1. Plotting of the solutions when there is no heat exchange: a) in the phase diagram; b) in the auxiliary plane (σ, G).

With the above assumptions, the process of displacing oil from a linear bed $0 \leqslant x \leqslant L$ with the cross-sectional area A is described by the equations of the balance of water and heat in the flow [4-6]:

$$\frac{\partial \sigma}{\partial \tau}+\frac{\partial G}{\partial X}=0, \quad \frac{\partial}{\partial \tau}[T(\sigma+b)]+\frac{\partial}{\partial X}[T(G+h)]+\varkappa(T-T_0)=0,$$
$$X=mAx/V_p, \quad \tau=Ut/V_p, \quad V_p=mAL, \quad 0 \leqslant X \leqslant 1, \tag{1}$$
$$\varkappa=\alpha V_p/mAU(c_1-c_2), \quad b=(c_2+c_3/m)/(c_1-c_2), \quad h=c_2/(c_1-c_2).$$

Here the dimensionless variables X and τ have the meaning of the volume of part of the bed from its inlet section x = 0 and the volume of the liquid pumped in, referred to the pore volume of the bed V_p.

The volume proportion of water in the flow G depends on the phase composition of the water—oil mixture. In single-phase and two-phase states we have, respectively:

$$G(\sigma, T)=\sigma, \quad T \geqslant T^*(\sigma), \tag{2}$$

$$\sigma(s, T)=sw+(1-s)\psi, \quad G(\sigma, T)=Fw+(1-F)\psi, \quad T<T^*(\sigma),$$
$$w=w(T), \quad \psi=\psi(T), \quad F(s, T)=\frac{f_1/\mu_1}{f_1/\mu_1+f_2/\mu_2}, \tag{3}$$
$$f_i=f_i(s, T), \quad \mu_i=\mu_i(T),$$

where F(s, T) is the proportion of aqueous phase in the flow.

We seek the solution of the system (1)-(3) with the initial and boundary conditions

$$\sigma=\sigma_0, \quad T=T_0(\tau=0, X>0); \quad \sigma=1, \quad T=T^0(\tau>0, X=0). \tag{4}$$

We put the points D and D^0 on the phase diagram in the plane (σ, T) in correspondence with the values of (σ_0, T_0) and $(1, T^0)$. We will assume that $T^0 > T^*(1)$, $T_0 < T^*(\sigma_0)$, $w(T_0) = 1$, $\psi(T_0) = 0$, i.e., that the points D^0 and D_0 are situated in the regions Δ' and Δ'', respectively, and that at the initial temperature of the bed T_0, water and oil are mutually insoluble; the magnitude of σ_0 coincides with the initial water saturation of the bed.

The solution of the problem (1)-(4) σ(x, τ), T(x, τ) is found efficiently by the method of characteristics. It is constructed from the final set of elements containing the sections of continuous change of the variables and their jumps (discontinuities). The sections of continuity of σ and T may belong to four types: 1) σ = 1, $\partial T/\partial X \leqslant 0$ for $T > T^*(1)$ (for the sake of brevity these sections will henceforth be called T_I-waves); 2) σ = const, $T = T_0$; 3) $\partial\sigma/\partial X \neq 0$, $T = T_0$ (σ-waves); 4) $\partial\sigma/\partial X \neq 0$, $\partial T/\partial X \leqslant 0$ for $T < T^*(\sigma)$ (T_{II}-waves).

Jumps may belong to one of three types: 1) jumps of σ and T with transition of single-phase flow into two-phase flow (denoted henceforth as J_{12}); 2) jumps of σ and T in two-phase flow (T-jumps); 3) jumps of σ for $T = T_0$ (σ-jumps).

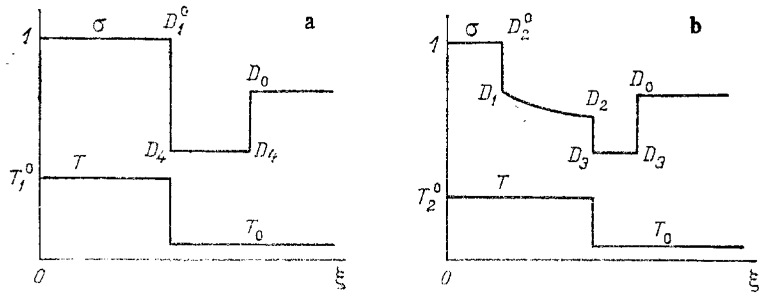

Fig. 2. Distributions of water content σ and of temperature T in the flow: a) in pseudopiston displacement; b) upon formation of a mixing zone.

On sections of continuity the system (1)-(3) is written in characteristic form [11]:

$$\frac{dX}{d\tau}=\xi_1=G'_\sigma,\quad \frac{d\sigma}{d\tau}=\frac{G'_T}{\xi_2-\xi_1}\left(\frac{dT}{d\tau}+\varkappa\frac{T-T_0}{\sigma+b}\right), \tag{5}$$

$$\frac{dX}{d\tau}=\xi_2=\frac{G+h}{\sigma+b},\quad \frac{dT}{d\tau}=-\varkappa\frac{T-T_0}{\sigma+b}. \tag{6}$$

In the jumps the integral laws of conservation have to be fulfilled:

$$\xi_j[\sigma]=[G],\quad \xi_j[T(\sigma+b)]=[T(G+h)],$$
$$[f]=f^+-f^-,\quad f^\pm(X,\tau)=f(X_j\pm 0,\tau),\quad \xi_j=dX_j/d\tau,$$

where X_j and ξ_j are dimensionless coordinates and speeds of the jump. In addition, in the construction of the jumps additional conditions of stability [11, 12] are used. In our case the unique solution is singled out by the condition that the number of characteristics per line of the jump $X = X_j(\tau)$ (whose characteristic speeds satisfy the inequalities $\xi_i^-\geqslant\xi_j$, $\xi_k^+\leqslant\xi_j$, $i, k = 1, 2$), be equal to three — analogously to the condition of evolution used in the theory of shock waves [11].

In constructing the solution, we first find its mapping into the auxiliary plane (σ, G), and in particular, to the distributions $\sigma(X, \tau)$ and $T(X, \tau)$ at each instant τ we put in correspondence some curve $G=\mathscr{L}(\sigma,\tau)$ called the "path" [4, 6] such that $\mathscr{L}(\sigma,\tau)= G(\sigma(X, \tau), T(X, \tau))$ for $0 \leqslant X < \infty$. Here the sections of continuity of σ and T are mapped into piecewise smooth arcs of the curve $\mathscr{L}(\sigma,\tau)$, the sections of constancy of σ and T into corner points, and jumps are represented by straight-line segments. Thus the path is a continuous curve connecting the points $D^0 = (1, G(1, T^0))$ and $D_0 = (\sigma_0, G(\sigma_0, T_0))$ of the plane (σ, G). On account of (5)-(7) the characteristic speeds ξ_1, ξ_2 and the speeds of the jumps ξ_j permit lucid graphic interpretation in the plane (σ, G) [6, 7]. Thus, $\xi_1(\sigma, T)$ coincides with the slope of the tangent to the curve $G(\sigma, T)$, $\xi_2(\sigma, T)$ with the slope of the secant passing through the point $P = (-b, -h)$ and called henceforth T-ray; analogously, the speed of the σ-jump ξ_j is equal to the slope of the straight line connecting the points $(\sigma^+, G(\sigma^+, T))$ and $(\sigma^-, G(\sigma^-, T))$, and the speed of the T-jump is equal to the slope of the T-ray passing through the point $(\sigma^\pm, G(\sigma^\pm, T^\pm))$. The principal property of the path is that the constant values of σ are transferred in the flow at the speeds $dX/d\tau$, coinciding at each instant τ with the slopes of the path $\beta(\sigma)=d\mathscr{L}(\sigma,\tau)/d\sigma$ in the plane (σ, G).

An advantage of the given method of constructing the solution is that the evolution of the path in time, and consequently also the structure of the solution, are fully established by elementary graphoanalytical means before the solution itself is constructed, after which the procedure of restoring the distributions $\sigma(X, \tau)$ and $T(X, \tau)$ in the flow is carried out automatically. Together with the path $G=\mathscr{L}(\sigma,\tau)$ it is expedient to construct simultaneously the mapping of the solution into the plane (σ, T) of the phase diagram, placing it above the (σ, G)-diagram (this circumstance was pointed out by P. G. Bedrikovetskii). Together with additional lucidity, such a method makes it possible to ascertain the temperatures T expended in mapping the T_I-waves into the plane (σ, G) because the latter are mapped into points of the straight line σ = G, independently of the values of T.

Let us first examine the principal solutions when there are no heat losses into the surrounding rocks ($\varkappa=0$). In this case the solution of the problem (1)-(4) is self-similar:

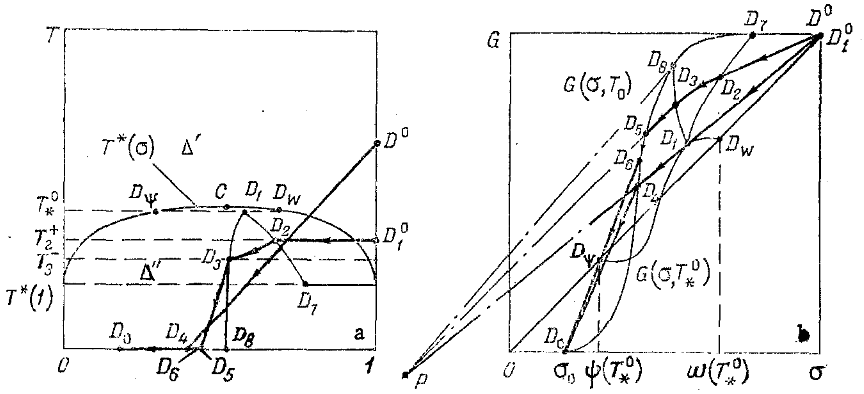

Fig. 3. Construction of the solutions taking heat exchange into account: a) in the phase diagram; b) in the auxiliary plane (σ, G).

$\sigma = \sigma(\xi)$, $T = T(\xi)$, $\xi = X/\tau$, so that the path $\mathscr{L}$ in the plane (σ, G) and in the phase diagram does not depend on the time, and $\xi = \beta(\sigma) = d\mathscr{L}(\sigma)/d\sigma$. Moreover, sections of type T_{II}-waves are impermissible, and on T_I-waves T = const. Figure 2 shows the distributions of $\sigma(\xi)$ and $T(\xi)$, and Fig. 1 shows the paths in the plane (σ, G) and in the plane (σ, T) for the two principal types of solution corresponding to the case of complete displacement of residual oil (when the initial water saturation of the bed is high). To the solution of the first type (Fig. 2a) corresponds to path $D_1^0D_4D_0$, to the solution of the second type (Fig. 2b) the path $D_2^0D_1D_2D_3D_0$. In the solution of the first type the jump J_{12} occurs along the T-ray passing through the point D_1^0, and in the solution of the second type it corresponds to the line segment $D_2^0D_1$ of the straight line touching the curve $G(\sigma, T_2^0)$ at the point D_1; here the T-jump is represented by the segment of the T-ray touching the curve $G(\sigma, T_2^0)$ at the point D_2. The remaining constructions are carried out in the known manner [4, 6]. Other self-similar solutions of both types and their constructions were dealt with in [3]. The solution of first type corresponds to the pseudopiston regime of displacement: the thermal front moves at the speed $\zeta_1 = (1 + h)/(1 + b) < 1$ (lower than the pumping speed), completely displacing the oil contained in the bed. As a result, a zone of increased oil content in the flow forms in the structure of the displacement front: the oil swell (D_4D_4 in Fig. 2a). When the temperature of the water that is forced in is lower ($T_2^0 < T_1^0$), a regime of displacement obtains, involving the formation of a zone of mixing of water and oil at $T = T_2^0$ (D_1D_2 in Fig. 2b) in front of the zone of hot water ($D_2^0D_2^0$) from which the oil is displaced completely. In this case the oil swell has a more complex structure. The temperature of transition from the first to the second regime, T_*^0 may be given the following graphic interpretation in the plane (σ, G): with $T^0 = T_*^0$ the curve $G(\sigma, T^0)$ touches the T-ray PD^0 (Fig. 3). From the constructions in Figs. 1 and 3 it is clear that $T_*^0 < T_C$, i.e., the temperature of transition to the pseudopiston regime of displacement is lower than the temperature of full mutual solubility of water and oil. When $T^0 > T_*^0$, hot water is an "ideal" solvent, and the process of displacement does not depend on the temperature T^0. This result is analogous to the known result for processes of displacement of oil by hydrocarbonaceous gas [13, 14] where the piston regime of displacement may be put into effect with incomplete mutual solubility of the displacing and the displaced liquids. However, whereas in displacement of oil by hydrocarbonaceous gases the compositions of the ideal displacing agents are determined exclusively by the conditions of phase equilibrium, in our case the temperature of transition to the pseudopiston regime of displacement is determined by the dependence of the hydrodynamic characteristics of the flow on the temperature, i.e., by the form of the function G(σ, T).

Let us now examine the effect of heat exchange on the structure of the displacement front. We will assume that the temperature of the pumped-in water is $T^0 > T_*^0$, and that the initial water saturation of the bed σ_0 is low (Fig. 3a). Since the rate of heat exchange is finite, the structure of the displacement front at the initial instant coincides with the structure of the self-similar solution for $\varkappa = 0$. The corresponding path in the plane (σ, G) has the shape of the curve $D^0D_4D_0$. As the thermal front $X = X_1(\tau)$ moves along, the temperature behind it drops, however, as long as the temperature on the front $T_1^-(\tau) \geqslant T_*^0$, its change does not affect the distribution of the water content σ. When $X \leqslant X_1(\tau)$

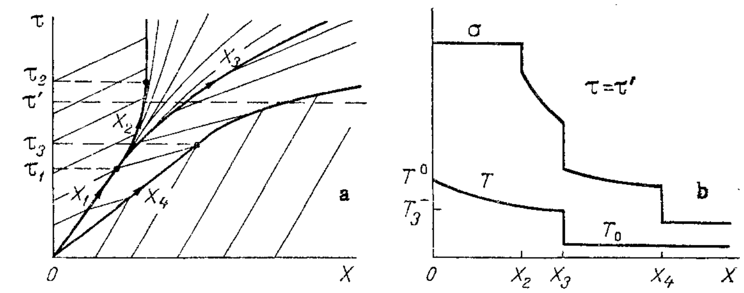

Fig. 4. Structure of the displacement front in case of heat exchange: a) trajectory of the jumps and characteristics of the first family; b) distributions of water content σ and of temperature T in the flow.

$$X_1(\tau) = \frac{1+h}{1+b}\tau, \quad T(X,\ \tau) = T_0 + (T^0 - T_0)\exp\left(-\frac{\varkappa X}{1+h}\right),$$
$$T_1^-(\tau) = T(X_1,\ \tau), \tag{8}$$

i.e., the temperature at each point of the heated zone of the bed depends solely on the coordinate and not on time. Equality $T_1^-(\tau) = T_*^0$ is attained with $\tau = \tau_1 = -(1 + b)\ln[(T_*^0 - T_0)/(T^0 - T_0)]/$. Further drop of the temperature on the thermal front leads to the formation of a two-phase zone of mixing of water and oil; the corresponding path assumes the shape of the curve $D^0D_2D_3D_5C_6D_0$ in the plane (σ, G) and of the curve $D^0D_1^0D_2D_3D_5D_6D_0$ in the plane (σ, T) (Fig. 3). We denote the boundaries of the mixing zone $X_2(\tau)$ and $X_3(\tau)$ ($X_2 < X_3$) and the boundary values of T and σ in it $T_2^+(\tau)$, $\sigma_2^+(\tau)$ and $T_3^-(\tau)$, $\sigma_3^-(\tau)$. When $X = X_2(\tau)$, the discontinuity applies only to the water content σ, and when $X = X_3(\tau)$, to σ and T simultaneously. The movement of the fronts $X_2(\tau)$ and $X_3(\tau)$ for $\tau > \tau_1$ is described by the equations

$$\frac{dX_2}{d\tau} = \xi_1(\sigma_2^+,\ T_2^+) = \frac{G(\sigma_2^+,\ T_2^+) - 1}{\sigma_2^+ - 1}, \quad \frac{dT_2^+}{d\tau} = -\varkappa\frac{T_2^+ - T_0}{\sigma_2^+ + b}; \tag{9}$$

$$\frac{dX_3}{d\tau} = \xi_1(\sigma_3^-,\ T_3^-) = \xi_2(\sigma_3^-,\ T_3^-), \quad \frac{dT_3^-}{d\tau} = -\varkappa\frac{T_3^- - T_0}{\sigma_3^- + b}; \tag{10}$$
$$X_2(\tau_1) = X_3(\tau_1) = X_1(\tau_1), \quad T_2^+(\tau_1) = T_3^-(\tau_1) = T_*^0,$$

which are integrated by separating the variables [3]. As a result we find the laws of motion of the jumps in parametric form:

$$X_2 = X_2(T_2^+), \quad \tau = \tau(T_2^+); \quad X_3 = X_3(T_3^-), \quad \tau = \tau(T_3^-), \tag{11}$$

where T_2^+ changes from T_*^0 to $T^*(1)$, and T_3^- from T_*^0 to 0.

On account of (9) the value $T_2^+ = T^*(1)$ is attained within finite time $(\tau_1 \leqslant \tau \leqslant \tau_2)$. From the region $0 \leqslant X \leqslant X_2(\tau_2)$ the oil is completely displaced. When $\tau > \tau_2$, the σ-jump is not moving, and in the mixing zone the oil does not dissolve in water. At this stage of displacement, more complete extraction of oil depends on reduction of its viscosity at higher temperatures. The values of σ and T in the mixing zone are efficiently found by numerical integration of the equations of the characteristics with the aid of the ordinary finite-difference corner schema. Thanks to the explicit separation of the jumps, the segment $X_2(\tau) \leqslant X \leqslant X_3(\tau)$ is preliminarily normalized, and this makes it possible to ascertain the volume step of a uniform network, and to choose the step in time in accordance with the condition of stability of the schema [8].

The influence of heat losses on the structure of the forward zone of displacement $X > X_3(\tau)$, not thermally affected, manifests itself in increased speed of propagation (beginning at some instant $\tau = \tau_3$) of the front of cold water (of the σ-jump). Let $\sigma_4^-(\tau)$ be the water content on the front, then (see Fig. 3) with

$$\sigma_4^-(\tau) = \sigma_4, \quad X_4(\tau) = \frac{G(\sigma_4,\ T_0) - G(\sigma_0,\ T_0)}{\sigma_4 - \sigma_0}\tau,$$

$$\tau_3 = \frac{G'_\sigma(\sigma_4, T_0) - (1+h)/(1+b)}{G'_\sigma(\sigma_4, T_0) - X_4(\tau)/\tau} > \tau_1. \tag{12}$$

With $\tau > \tau_3$ $X_4(\tau)$ and $\sigma_4^-(\tau)$ we find in the quadratures from the relations

$$\frac{dX_4}{d\tau} = \frac{G(\sigma_4^-, T_0) - G(\sigma_0, T_0)}{\sigma_4^- - \sigma_0}, \quad \frac{X_4(\tau) - X_3(\tau')}{\tau - \tau_1} = \xi_1(\sigma_3^+, T_0), \tag{13}$$

$$\tau'(\tau_3) = \tau_1, \quad \sigma_4^-(\tau) = \sigma_3^+(\tau'), \quad \sigma_3^+(\tau_1) = \sigma_4$$

with a view to (12); here, σ_3^+ is the water content ahead of the mixing zone.

In the zone $X_3(\tau) < X < X_4(\tau)$ the temperature is constant ($T = T_0$), and therefore in accordance with (5) the constant values of σ are transferred along the characteristic $dX/d\tau = \xi_1(\sigma, T_0)$. The trajectories of the jumps and characteristics of this family in the plane (X, τ) are shown in Fig. 4a; in Fig. 4b the distributions of σ and T for $\tau_3 < \tau < \tau_2$ are plotted. Other examples of solutions corresponding to the entire range of values of σ_0 were dealt with in [3].

NOTATION

x, coordinate; t, time; T, local temperature of the bed; T_0, initial temperature of the bed and the surrounding rocks; T°, temperature of the pumped-in water; s, saturation of the aqueous phase; F, proportion of aqueous phase in the flow; σ, water content; G, proportion of water in the flow; w, ψ, volume contents of water in the aqueous and oil phases, respectively; f_1, f_2, relative phase permeabilities for aqueous and oil phases, respectively; μ_1, μ_2, viscosities of the aqueous and oil phases, respectively; c_1, c_2, c_3, heat capacities of water, oil, and rock, respectively; m, porosity; V, volume flow rate; α, heat-transfer coefficient of the bed with the surrounding rocks per unit volume of the bed; $T = T^*(\sigma)$, binodal curve of the phase diagram; T_C, temperature of full mutual solubility of water and oil; X, τ, dimensionless coordinate and time, respectively; ξ_i, dimensionless characteristic speeds; ξ_j, dimensionless speed of the jump.

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HELICAL FLOW OF A NONLINEARLY VISCOPLASTIC LIQUID IN AN ANNULAR CHANNEL

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This article presents analytical expressions describing the flow of a generalized nonlinearly viscoplastic liquid in a concentric annular channel under complex shear. The results of numerical calculations are analyzed.

The theoretical and experimental investigation of helical flow in pipes was dealt with by many authors of whom we mention [1-5]. Rivlin [1], e.g., obtained some general relations for the helical flow of a generalized non-Newtonian liquid in an annular channel. The experimental verification of the generalized regularity of flow under conditions of complex shear was carried out by Vinogradov et al. [2]. Coleman et al. [3] obtained general relations for the distribution of longitudinal and angular velocities in a channel and the flow rate of a generalized non-Newtonian liquid. However, numerical calculations with their aid were carried out for the first time by Prokunin et al. [4] for an exponential liquid. All these authors dealt with liquids that do not have a limit shear stress. As far as the few existing investigations dealt with viscoplastic liquids, only the qualitative aspect of the problem was studied, and this does not make it possible to carry out numerical calculations and their corresponding analysis. For instance, Myasnikov [5] obtained a phase diagram expressing qualitatively the nature of profiles of longitudinal and angular velocity of the helical flow of a Bingham—Shvedov liquid. The present authors obtained a closed system of equations in dimensionless form enabling them to calculate the principal characteristics of the helical flow of a nonlinearly viscoplastic liquid in an annular channel, both for the normal and the inverse hydraulic problem.

We examine laminar steady-state flow in a concentric annular channel formed by two long cylinders with radii a and b (a < b), with constant pressure gradient $-P = -\Delta p/l$, acting along the cylinder axis z. The inner cylinder rotates at constant angular velocity Ω_0. As rheological model we use the generalized model of Shul'man which is adequate for the rheological behavior of various paint and varnish compositions, pulps, foodstuffs, cement and clay suspensions, and a number of other non-Newtonian media:

$$\eta\dot{\gamma} = (\tau^{1/n} - \tau_0^{1/n})^m, \quad \tau > \tau_0,$$

$$\dot{\gamma} = 0, \quad \tau \leqslant \tau_0. \tag{1}$$

$$\dot{\gamma} = \sqrt{(u_r')^2 + (r\omega_r')^2}, \tag{2}$$

τ is the tangential stress intensity,

$$\tau = \sqrt{(\tau_{rz})^2 + (\tau_{r\theta})^2}. \tag{3}$$

Expressions (1)-(3) are written with a view to the axisymmetric nature of the helical flow. For the stress components τ_{rz} and $\tau_{r\theta}$ the following relations ensuing from the equations of equilibrium of an element of the liquid [1] are correct:

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